

# Quasi-elastic neutrino charged-current scattering of medium-heavy nuclei: $^{40}\text{Ca}$ and $^{40}\text{Ar}$

A. Butkevich

Institute for Nuclear Research, Moscow

20 December 2011

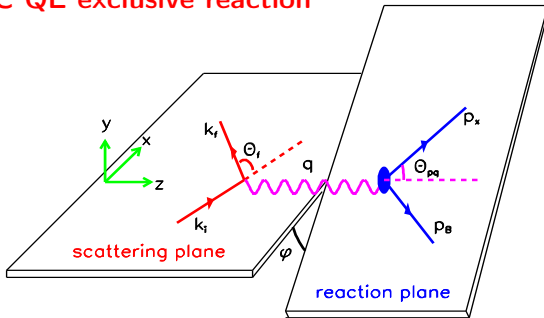
*Femilab, PPD/Neutrino Physics Discussion*

# Motivation

- (★) To make neutrino detectors fully active, highly segmented and fine-grained **scintillator** ( $^{12}\text{C}$ ) is used as target and detecting material.
- (★) **Liquid Argon TPC** are one of the most perspective technologies for next generation of neutrino detector **ICARUS**, **MickroBooNe**, **LNBE**.
- (★) There are no the cross section data about quasi-elastic electron and neutrino scattering off **Argon**. On the other hand there are precision data for electron scattering on  $^{40}\text{Ca}$  target. **The structures of  $^{40}\text{Ca}$  and  $^{40}\text{Ar}$  are almost identical**.
- (★) Therefore it is reasonable to calculate cross sections of QE electron scattering off  $^{40}\text{Ca}$  target to test models predictions against  $e^{40}\text{Ca}$  data.
- (★) In long baseline neutrino experiments the near and far detectors **are not necessary of the same target material**. It is important to compare the CCQE cross sections for neutrino scattering off  $^{12}\text{C}$ ,  $^{16}\text{O}$ , and  $^{40}\text{Ar}$  targets.

## Formalism of the CCQE Scattering

## Lepton CC QE exclusive reaction



$$l(k_i) + A(p_A) \rightarrow l'(k_f) + N(p_x) + B(p_B),$$

$$\frac{d^5 \sigma^{el}}{d\varepsilon_f d\Omega_f d\Omega_x} = R \frac{|p_x| \varepsilon_x}{(2\pi)^3} \frac{\varepsilon_f}{\varepsilon_i} \frac{\alpha^2}{Q^4} L_{\mu\nu}^{(el)} W^{\mu\nu(el)}$$

$$\frac{d^5 \sigma^{cc}}{d\varepsilon_f d\Omega_f d\Omega_x} = R \frac{|p_x| \varepsilon_x}{(2\pi)^5} \frac{|k_f|}{\varepsilon_i} \frac{G^2 \cos^2 \theta_c}{2} L_{\mu\nu}^{(cc)} W^{\mu\nu(cc)},$$

$L_{\mu\nu}$  and  $W^{\mu\nu}$  are lepton and hadronic tensors.  $\varepsilon_f$ ,  $\Omega_f$  are energy and solid angle of the scattered lepton.  $\Omega_x$  is solid angle for ejectile nucleon momentum.

# Formalism

The weak CC **hadronic tensor**  $W_{\mu\nu}$  is given by bilinear product of the transition matrix elements of the nuclear CC operator  $J_\mu$  between the initial nucleus state  $|A\rangle$  and the final state  $|B_f, p_x\rangle$  as

$$W_{\mu\nu} = \sum \langle B_f, p_x | J_\mu | A \rangle \langle A | J_\nu | B_f, p_x \rangle$$

It is also useful to define a **reduced cross section**

$$\sigma_{red} = \frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} / K \sigma_{lN},$$

where  $K$  is phase-space factor for lepton scattering, and  $\sigma_{lN}$  is corresponding elementary cross section for the lepton scattering from moving free nucleon.

$\sigma_{red}$  should be similar for electron and neutrino scattering (apart from small differences due to coulomb corrections and FSI effects for electron and neutrino induced reactions).

In the **inclusive reactions** only the outgoing lepton is detected and the differential cross sections can be written as

$$\frac{d^3\sigma^{el}}{d\varepsilon_f d\Omega_f} = \frac{\varepsilon_f}{\varepsilon_i} \frac{\alpha^2}{Q^4} L_{\mu\nu}^{(el)} \mathcal{W}^{\mu\nu(el)},$$

$$\frac{d^3\sigma^{cc}}{d\varepsilon_f d\Omega_f} = \frac{1}{(2\pi)^2} \frac{|k_f|}{\varepsilon_i} \frac{G^2 \cos^2 \theta_c}{2} L_{\mu\nu}^{(cc)} \mathcal{W}^{\mu\nu(cc)}$$

where  $\mathcal{W}^{\mu\nu}$  is inclusive hadronic tensor.

# Nuclear matrix element

- We describe the lepton-nucleon scattering in the Impulse Approximation (IA), in which only one nucleon of the target is involved in reaction and the nuclear current is written as the sum of single-nucleon currents.

$$\langle p, B | J^\mu | A \rangle = \sum \int d^3r \exp(it \cdot r) \bar{\Psi}^{(-)}(p, r) \Gamma^\mu \Phi(r),$$

where  $\Gamma^\mu$  is the vertex function,  $\Phi$  and  $\Psi^{(-)}$  are relativistic bound-state and outgoing wave functions.

- Electromagnetic current vertex in CC2 scheme

$$\Gamma_V^\mu = F_V(Q^2) \gamma^\mu + i \sigma^{\mu\nu} q_\nu F_M(Q^2) / 2m \quad (2)$$

- The single-nucleon charged current has  $V-A$  structure  $J^\mu = J_V^\mu + J_A^\mu$  and for  $\Gamma^\mu = \Gamma_V^\mu + \Gamma_A^\mu$  we use a free nucleon vertex function

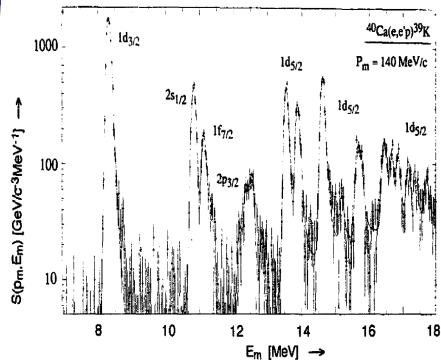
$$\Gamma_V^\mu = F_V(Q^2) \gamma^\mu + i \sigma^{\mu\nu} q_\nu F_M(Q^2) / 2m$$

and the axial current vertex function

$$\Gamma_A^\mu = F_A(Q^2) \gamma^\mu \gamma_5 + F_P(Q^2) q^\mu \gamma_5.$$

We use the MMD approximation [P.Mergell et al (1996)] of the vector nucleon form factors and the dipole approximation for the axial  $F_A$  and pseudoscalar  $F_P$  form factors.

# Model



The typical excitation energy spectrum of  $^{40}\text{Ca}$  measured in NIKHEF experiment in missing energy range  $0 < \epsilon_m < 23 \text{ MeV}$ . The knock-out from  $1d_{3/2}$ ,  $2s_{1/2}$ ,  $1d_{5/2}$ ,  $1f_{7/2}$ , and  $2p_{3/2}$  are observed.

The  $\sigma_{red}$  cross sections in  $^{40}\text{Ca}(e, e'p)^{39}\text{K}$  reaction were measured in the follow experiment:

Tokyo: K.Nakamura et al. Nucl.Phys.A271(1976)221, Saclay: M.Bernheim et al. Nucl.Phys. A375(1982)381, NIKHEF:G.J.Kramer et al. Phys.Lett. B227(1989)199, Nucl.Phys. A679(2001)267

In IPSM the model space for  $^{40}\text{Ca}$  consists of 6 states for proton(neutron):

$1d_{3/2}(4)$ ,  $2s_{1/2}(2)$ ,  $1d_{5/2}(6)$ ,  $1p_{1/2}(2)$ ,  $1p_{3/2}(4)$ ,  $1s_{1/2}(2)$ . The observed  $1f_{7/2}$  and  $2p_{1/2}$  is result of long-range nucleon correlations.

## Nucleon momentum distribution measured in Saclay experiment

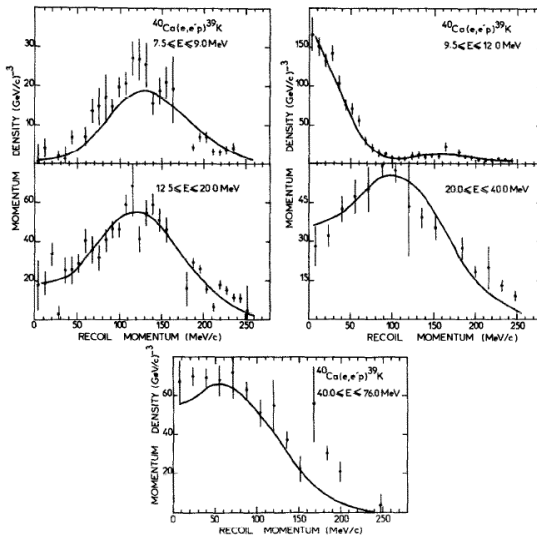


Fig. 14. Momentum distribution from  $^{40}\text{Ca}(e, e'p)^{39}\text{K}$ ; (a)  $7.5 \leq E \leq 9 \text{ MeV}$ , (b)  $9.5 \leq E \leq 12 \text{ MeV}$ , (c)  $12.5 \leq E \leq 20 \text{ MeV}$ , (d)  $20 \leq E \leq 40 \text{ MeV}$  and (e)  $40 \leq E \leq 70 \text{ MeV}$ . The solid line represents the DWIA calculation.



The value of the spectroscopic factors for  $^{40}\text{Ca}$  orbitals were deduced from  $^{40}\text{Ca}(e, e'p)^{39}\text{K}$  reactions in *G.J.Kramer Ph.D. thesis (1991), G.J. Kramer et al. Phys.Lett. B227(1989)199, J.P.McDermott PRL65(1990)1991, C.Mahaux, R.Sator Nucl.Phys.A484 (1988)205, J.M. Udias et al. PRC48(1993)2731 and PRC51(1995)3246.*

The shape of momentum distribution is related to the shape of bound-state wave and the strength is proportional to the spectroscopic factors of the shells.

In this calculation we use the shell occupancy for  $^{40}\text{Ca}$

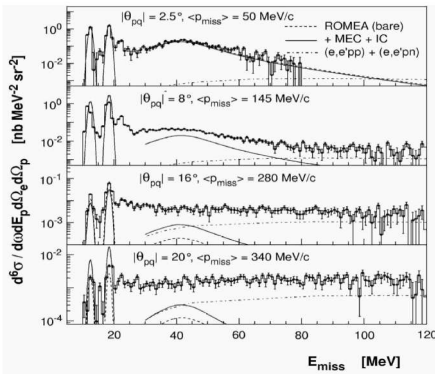
$S(1d_{3/2}) = 0.82, S(2s_{1/2}) = 0.845, S(1d_{5/2}) = 0.8, S(1p_{1/2}) = 0.95, S(1p_{3/2}) = 0.95, S(1s_{1/2}) = 1,$

for  $^{40}\text{Ar}$   $S(1d_{3/2}) = 0.845, S(2s_{1/2}) = 0.845, S(1d_{5/2}) = 0.8, S(1p_{1/2}) = 0.95, S(1p_{3/2}) = 0.95, S(1s_{1/2}) = 1$  for proton and

$S(1f_{7/2}) = 0.82, S(1d_{3/2}) = 0.82, S(2s_{1/2}) = 0.85, S(1d_{5/2}) = 0.8, S(1p_{1/2}) = 0.95, S(1p_{3/2}) = 0.95, S(1s_{1/2}) = 1$  for neutron.

Average occupancy of nuclear shells  $\bar{S} = 0.87$  and 13% of the missing strength can be attributed to the SRC(NN pair). Note that the calculation for the  $1d_{3/2}$  and  $2s_{1/2}$  states include the incoherent contributions of  $1f_{7/2}$  and  $2p_{1/2}$  states.

## Shell occupancy for Oxygen:



$$S(P_{1/2})=0.7$$

$$S(P_{3/2})=0.66$$

$$S(S_{1/2})=1$$

Average occupancy of nuclear shells  $\overline{S}=0.75$

[supported by JLab measurement K. G. Fissum et al. (2005)]

Missing strength of 25% can be attributed to SRC

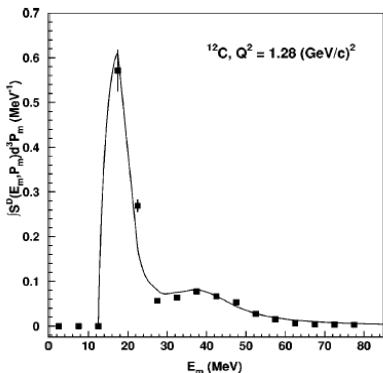
## Shell occupancy for Carbon:

$$S(P_{3/2})=0.84$$

$$S(S_{1/2})=1$$

Average occupancy of nuclear shells  $\bar{S}=0.89$

supported by JLab measurement  
[D. Dutta et al. (2003), J.J.Kelly (2004)]



Missing strength of 11% can be attributed to SRC.

JLab measurement  
[D. Rohe et al. (2006)]

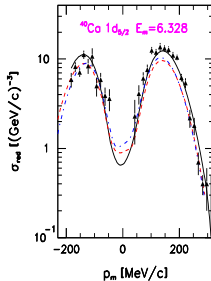
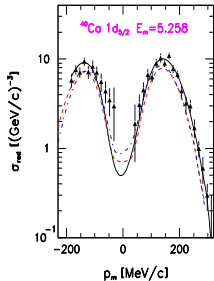
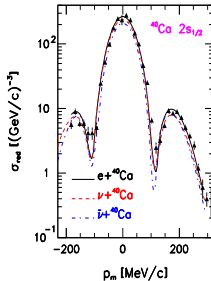
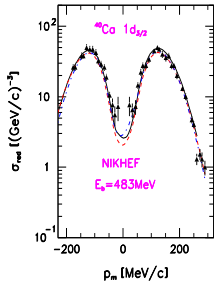
observed approximately 0.6 proton in a region with

$p_m > 240$  MeV and  $E_m > 50$  MeV.

- In independent particle shell model the relativistic bound-state functions  $\Phi$  are obtained within the Hartree–Bogolioubov approximation in the  $\sigma - \omega$  model [B.Serot et al. 1986].
  - ★ In our calculations the spectral function  $P_{HM}$  incorporates 13% of the total normalization of the spectral function.
- In the RDWIA the final state interaction between the outgoing nucleon and the residual nucleus is taking into account and the ejectile wave function  $\Psi$  is a solution of a Schrödinger equation containing equivalent central and spin-orbit potentials, which are functions of the scalar and vector potentials  $S$  and  $V$ , and are energy dependent.
- In Fermi Gas model for Calcium and Argon we use  $p_F=249$  MeV/c and  $\epsilon=33$  MeV. The RFGM does not account for nuclear shell structure, FSI effect, and the presence of NN-correlations.

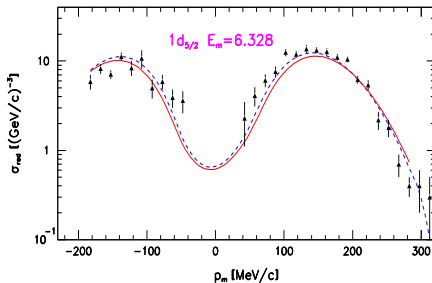
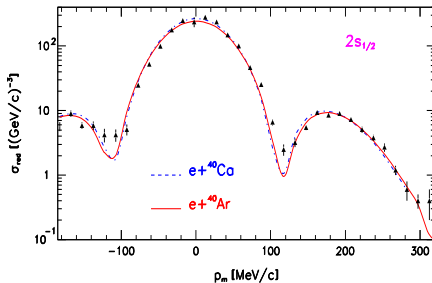
# RESULTS

# Reduced cross sections



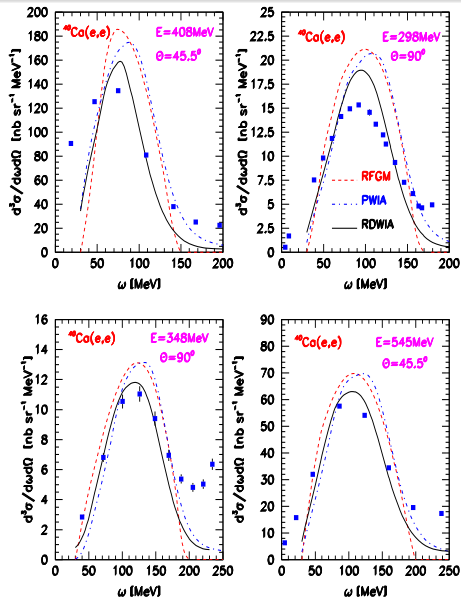
Comparison of the RDWIA calculations for electron, neutrino, and antineutrino reduced cross sections for the removal of nucleon from  $1d_{3/2}$ ,  $2s_{1/2}$  and  $1d_{5/2}$  shells of  $^{40}\text{Ca}$  as a function of missing momentum. The cross sections were calculated for NIKHEF [G.J.Kramer (1991)] kinematics:  $\varepsilon_i = 483$  MeV, and  $T_x = 100$  MeV. Negative values of  $p_m$  correspond to  $\phi = \pi$  and positive ones to  $\phi = 0$ .

# Reduces cross sections



Comparison of the RDWIA calculations for electron reduced cross sections for the removal of nucleon from  $2s_{1/2}$  and  $1d_{5/2}$  shells of  $^{40}\text{Ca}$  and  $^{40}\text{Ar}$  as a function of missing momentum. The cross sections were calculated for NIKHEF [G.J.Kramer (1991)] kinematics:  $\epsilon_i = 483$  MeV, and  $T_x = 100$  MeV. Negative values of  $p_m$  correspond to  $\phi = \pi$  and positive ones to  $\phi = 0$ .

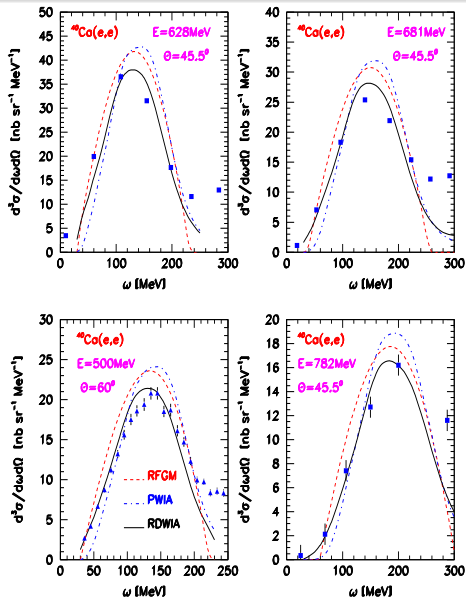
# Inclusive cross sections



Inclusive cross section versus the energy transfer  $\omega$  for electron scattering on  $^{40}\text{Ca}$ . The data are from MIT Bates for electron beam energy  $E_e = 408, 545 \text{ MeV}$  and scattering angle  $\theta = 45.5^\circ$  and for energy  $E_e = 298, 348 \text{ MeV}$  and  $\theta = 90^\circ$ . The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations.

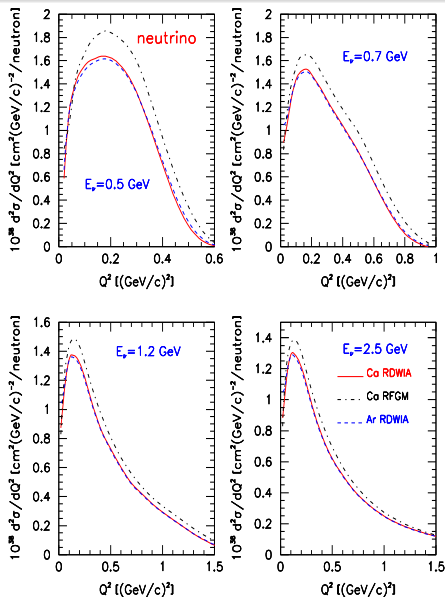


# Inclusive cross sections



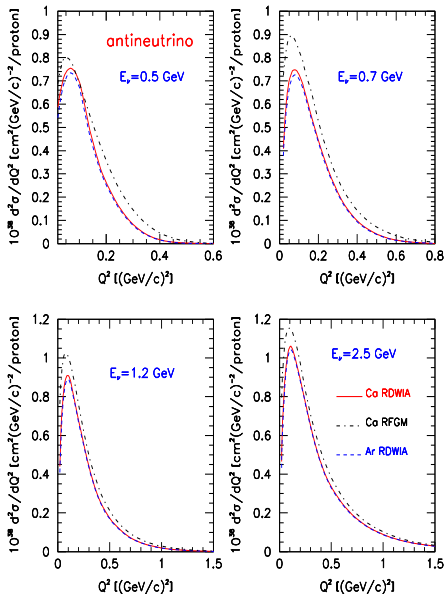
Inclusive cross section versus the energy transfer  $\omega$  for electron scattering on  $^{40}\text{Ca}$ . The data are from MIT Bates for electron beam energy  $E_e=628$ , 681, and 782 MeV and scattering angle  $\theta=45.5^\circ$  and from SLAC for energy  $E_e=500$  MeV and  $\theta=60^\circ$ . The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations.

# Inclusive cross sections



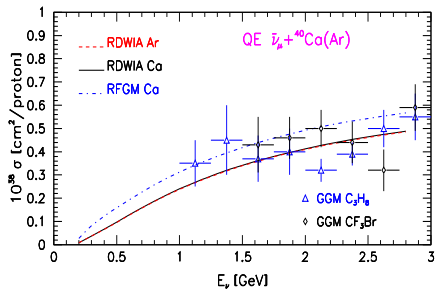
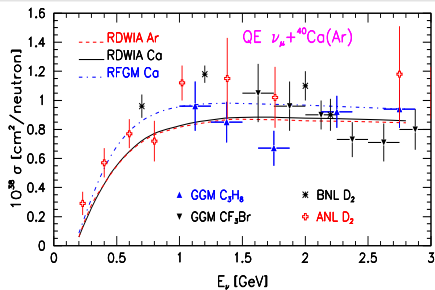
Inclusive cross section per neutron versus  $Q^2$  for neutrino scattering on  $^{40}\text{Ca}$  and  $^{40}\text{Ar}$  and for four values of incoming neutrino energy:  $E_\nu = 0.5, 0.7, 1.2$ , and  $2.5$  GeV. The solid(dashed) line is the RDWIA calculation for  $^{40}\text{Ca}$  ( $^{40}\text{Ar}$ ) and dashed-dotted line is RFGM calculations for  $^{40}\text{Ca}$ .

# Inclusive cross sections



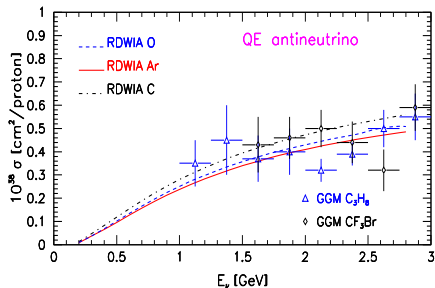
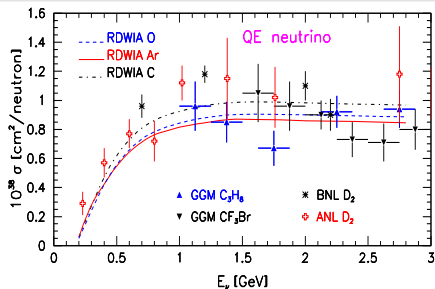
Inclusive cross section per proton versus  $Q^2$  for antineutrino scattering on <sup>40</sup>Ca and <sup>40</sup>Ar and for four values of incoming neutrino energy:  $E_\nu = 0.5, 0.7, 1.2$ , and  $2.5$  GeV. The solid(dashed) line is the RDWIA calculation for <sup>40</sup>Ca (<sup>40</sup>Ar) and dashed-dotted line is RFGM calculations for <sup>40</sup>Ca.

# Total cross sections



Total cross section for CCQE scattering of muon neutrino (per neutron, upper panel) and antineutrino (per proton, lower panel) on  ${}^{40}\text{Ca}$  and  ${}^{40}\text{Ar}$  as functions of incoming (anti)neutrino energy. The solid and dashed lines are respectively the RDWIA results for scattering on Calcium and Argon. The dashed-dotted line is the RFGM result for scattering on Calcium. These results were obtained with  $M_A = 1.032$  GeV.

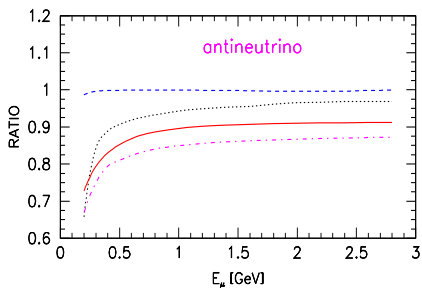
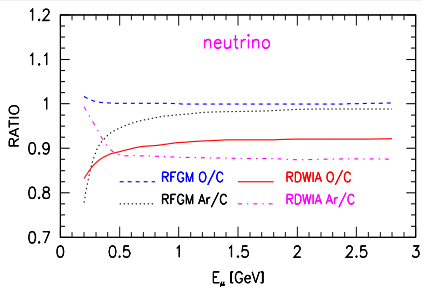
# Total cross sections



Total cross section for CCQE scattering of muon neutrino (per neutron, upper panel) and antineutrino (per proton, lower panel) on <sup>40</sup>Ar (solid line), <sup>16</sup>O (dashed line), and <sup>12</sup>C (dashed-dotted line) as functions of incoming (anti)neutrino energy. These results were obtained with  $M_A = 1.032$  GeV.

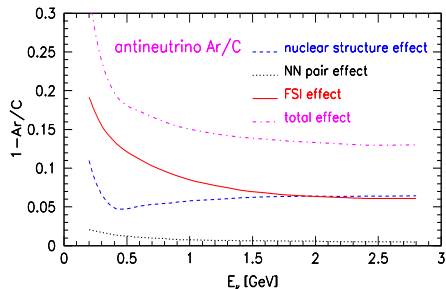
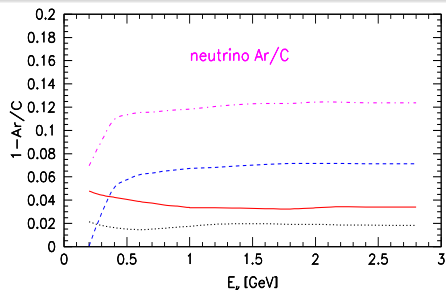
# DISCUSSION

# Ratio $O/C$ and $Ar/C$



Ratio of the total cross section per neutron/proton  $R=O/C$  and  $R=Ar/C$  versus neutrino energy for neutrino (top panel) and antineutrino (bottom panel) scattering off  $^{40}Ar$ ,  $^{16}O$ , and  $^{12}C$ . The solid and dotted-dashed lines are the RDWIA results while dotted and dashed lines are the RFGM results, respectively.

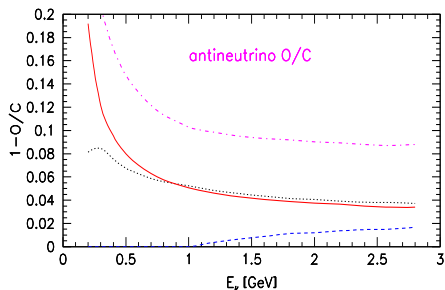
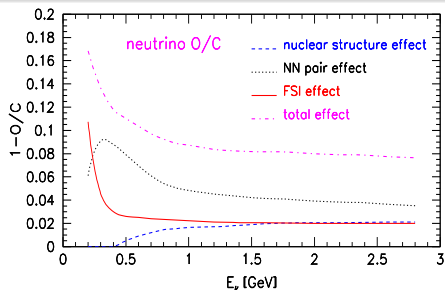
# Ratio Ar/C



The difference between total cross section (per neutrino/proton) for neutrino (top panel) and antineutrino (bottom panel) scattering off  $^{40}\text{Ar}$  and  $^{12}\text{C}$  ( $1 - \text{Ar/C}$ ) due to different nuclear structures (dashed line), correlated NN-pair contributions (dotted line), and FSI effects (solid line). The dashed-dotted line is total of the three effects.

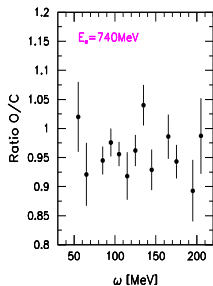
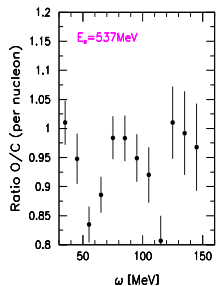
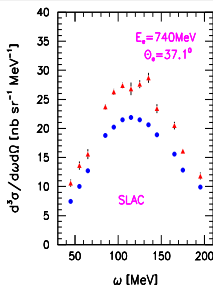
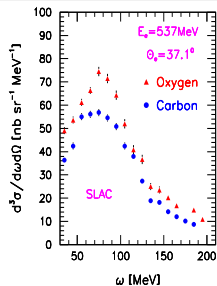


# Ratio O/C



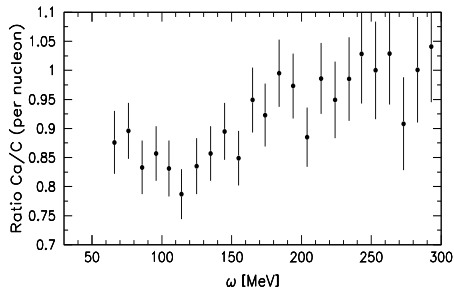
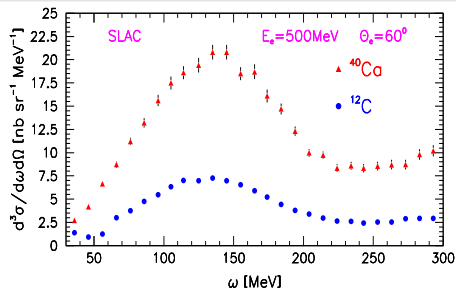
The difference between total cross section (per neutrino/proton) for neutrino (top panel) and antineutrino (bottom panel) scattering off  $^{16}\text{O}$  and  $^{12}\text{C}$  ( $1-O/C$ ) due to different nuclear structures (dashed line), correlated NN-pair contributions (dotted line), and FSI effects (solid line). The dashed-dotted line is total of three effects.

# Ratio O/C for electron scattering



Inclusive cross section (top panel) and the ratio  $R=O/C$  per proton (bottom panel) versus energy transfer for electron scattering off  $^{16}\text{O}$  and  $^{12}\text{C}$  measured in SLAC experiment J.S. O'Connell et al. PRC 35,1063(1987) for electron beam energies  $E_e = 537, 740$  MeV and  $\theta = 37.1^\circ$ .

# Ratio Ca/C for electron scattering



Inclusive cross section (top panel) and the ratio  $R=O/C$  per proton (bottom panel) versus energy transfer for electron scattering off  $^{40}\text{Ca}$  and  $^{12}\text{C}$  measured in SLAC experiment R.R. Whitney et al. PRC 9,2230(1974) for electron beam energy  $E_e = 500 \text{ MeV}$  and  $\theta = 60^\circ$ .

# CONCLUSIONS

QE CC  $\nu(\bar{\nu})^{40}\text{Ca}(\text{Ar})$  cross sections were studied in different approaches.

- The reduced cross sections for neutrino and electron scattering off  $^{40}\text{Ca}$  were tested against  $^{40}\text{Ca}(e, e'p)$  data.
  - ★ In RDWIA the cross sections for  $\nu(\bar{\nu})$  scattering are similar to those of electron scattering and in a good agreement with data (apart from small differences due to coulomb correction).
- The inclusive and cross sections were tested against  $^{12}\text{Ca}(e, e')$  scattering data.
  - ★ In the peak region RFGM overestimates the value of inclusive cross section at low momentum transfer. The discrepancy with data decreases as momentum transfer increases.
  - ★ The RDWIA result is in agreement with data with accuracy 10-15%.

- The inclusive  $d\sigma/dQ^2$  and total cross sections per neutron/proton for  $\nu(\bar{\nu})^{40}\text{Ca}(\text{Ar})$  interaction were calculated in the RFGM and RDWIA approaches and compared with ones for (anti)neutrino scattering on Oxygen and Carbon.
  - ★ In the RFGM the cross sections per nucleon for Carbon are practically similar to those for Oxygen and Argon.
  - ★ In the RDWIA the cross sections for Oxygen and Argon are lower than those for Carbon and the difference is decreases as neutrino energy increases.
  - ★ The nuclear structure effects can be the main source of the difference between the total cross section (per neutron/proton) for (anti)neutrino scattering off off light and (medium)heavy nuclei.